Write $u_{xx} = -|u_{xx}|, u_{yy} = -|u_{yy}|$. Then $y^2 u_{xx} + x^2 u_{yy} + 2xyu_{xy} = -y^2|u_{xx}| - x^2|u_{yy}| + 2xyu_{xy}$ and

$$-y^2|u_{xx}|-x^2|u_{yy}|+2xyu_{xy} < -y^2|u_{xx}|-x^2|u_{yy}|+2xy\sqrt{|u_{xx}||u_{yy}|} = -(y\sqrt{|u_{x}}}$$

so that again $y^2 u_{xx} + x^2 u_{yy} + 2xyu_{xy}$ is negative. This is in contradiction with $f > 0$ so that the maximum must be in the boundary. Now, for $f \geq 0$ we can do as before, and consider $v(x, y) = u(x, y) + \frac{\epsilon}{4}(x^2 + y^2)$. In this case

$$y^2 v_{xx} + x^2 v_{yy} + 2xyv_{xy} = f + \epsilon \tag{30}$$

and we can proceed exactly as before.