Math 255: Algebraic Curves

Bernd Sturmfels, UC Berkeley, Fall 2011
Homework # 4, due Tuesday, October 11

1. Let c denote the circumference of the ellipse defined by $2x^2 + 3y^2 = 5$. Compute c numerically up to 15 digits of accuracy. Express c as an integral $\int_\gamma f dg$ of a holomorphic differential on a Riemann surface.

2. Let C be the quartic in $\mathbb{P}^2$ defined by $x^4+y^4 = z^4$. Find a meromorphic differential on C and compute the corresponding canonical divisor.

3. Find an irreducible homogeneous polynomial $P(x, y, z)$ of degree 6 such that the curve $\{P(x, y, z) = 0\}$ has precisely 10 singular points in $\mathbb{P}^2$.

4. Let C be a smooth curve in complex projective 3-space $\mathbb{P}^3$ that is the intersection of two surfaces of degree d and e. What is the genus of C?

5. *[Kirwan 6.1]* Show that the integral of an exact holomorphic differential along a closed piecewise-smooth path on a Riemann surface S is 0. Deduce that the homolomorphic differential η on $\mathbb{C}/\Lambda$ is not exact.

6. *[Kirwan 6.15]* Show that any two nonzero holomorphic differentials on a compact connected Riemann surface are constant multiples of each other.

7. True or false: If D is a divisor on a nonsingular curve in $\mathbb{P}^2$ then the function $\mathbb{N} \to \mathbb{N}$, $m \mapsto l(mD)$ is given by a polynomial in m.

8. *[Fulton 8.30]* Given a nonsingular curve in $\mathbb{P}^2$, suppose that D and D' are divisors whose sum $D + D'$ is a canonical divisor. Then

$$l(D) - l(D') \;=\; \frac{1}{2}(\deg(D) - \deg(D')).$$