points. A "convex" arrangement is an arrangement whose dual set of points lies in convex position.

It is easy to see that the vertical height of a triangle is the minimum length of a vertical segment that connects a vertex of the triangle to the line supporting the opposite edge. This follows from the convexity of a triangle, and indeed that segment always lies inside the triangle. We now specify (in terms of the dual representation) the vertical distance between the intersection of two lines to a third line of $\mathcal{A}$ (in the primal representation). Refer to Figure 1. The equation of ℓ_i is

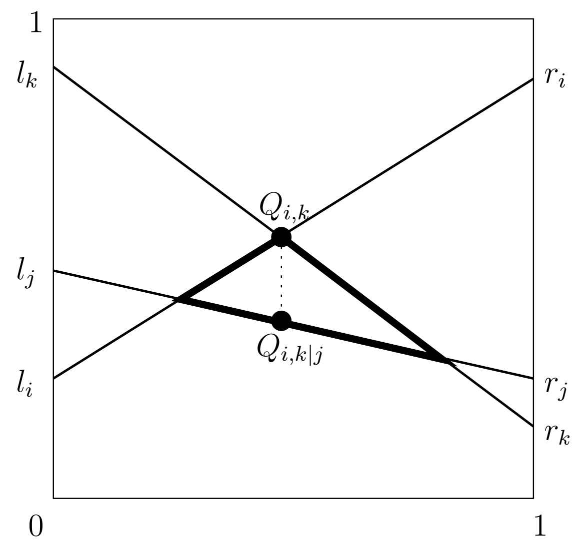

Fig. 1. Vertical distance

$y = (r_i - l_i)x + l_i$. We compute the distance between $Q_{i,k} = (x_{i,k}, y_{i,k})$, the intersection point of ℓ_i and ℓ_k, and $Q_{i,k|j} = (x_{i,k}, y_{i,k|j})$, the vertical projection of $Q_{i,k}$ on ℓ_j. A simple calculation shows that $Q_{i,k} = \left(\frac{l_k - l_i}{(l_k - l_i) - (r_k - r_i)}, \frac{l_k r_i - l_i r_k}{(l_k - l_i) - (r_k - r_i)}\right)$. By substituting $x_{i,k}$ in the equation of ℓ_j we find that $y_{i,k|j} = \frac{r_j(l_k - l_i) - l_j(r_k - r_i)}{(l_k - l_i) - (r_k - r_i)}$. Finally,

$$\mathrm{Dist}(Q_{i,k}, Q_{i,k|j}) = |y_{i,k} - y_{i,k|j}| = \left| \frac{r_i(l_k - l_j) - r_j(l_k - l_i) + r_k(l_j - l_i)}{(l_k - l_i) + (r_i - r_k)} \right|$$

$$= 4 \,\mathrm{abs}\left(\frac{\frac{1}{2} \begin{vmatrix} l_i & r_i & 1 \\ l_j & r_j & 1 \\ l_k & r_k & 1 \end{vmatrix}}{2((l_k - l_i) + (r_i - r_k))} \right).$$

The numerator of the last term is the area of the triangle defined by the points P_i, P_j, and P_k. In case P_i and P_k are in monotone decreasing position, the