By the two tangent theorem we have

$$AT_1 = AT_1'' = AP - PT_1'',$$
$$BT_1 = BT_1' = BP - PT_1',$$

so that

$$AB = AT_1 + BT_1 = AP + BP - PT_1'' - PT_1'.$$

Since $PT_1'' = PT_1'$,

$$AB = AP + BP - 2PT_1'.$$

In the same way

$$CD = CP + DP - 2PT_3'.$$

Adding the last two equalities yields

$$AB + CD = AC + BD - 2T_1'T_3'.$$

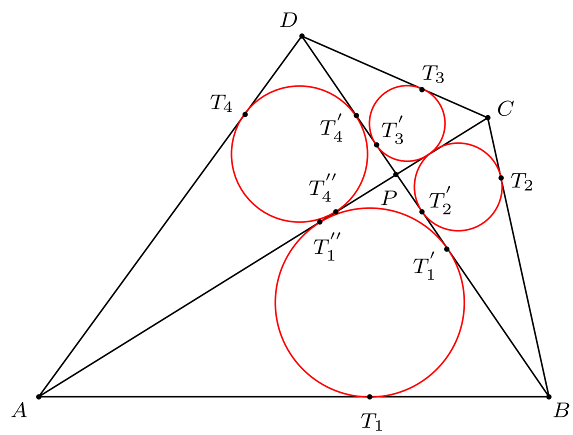

Figure 4. Tangency points of the four incircles

In the same way we get

$$BC + DA = AC + BD - 2T_2'T_4'.$$

Thus

$$AB + CD - BC - DA = -2\left(T_1'T_3' - T_2'T_4'\right).$$

The quadrilateral has an incircle if and only if $AB + CD = BC + DA$. Hence it is a tangential quadrilateral if and only if

$$T_1'T_3' = T_2'T_4' \quad \Leftrightarrow \quad T_1'T_2' + T_2'T_3' = T_2'T_3' + T_3'T_4' \quad \Leftrightarrow \quad T_1'T_2' = T_3'T_4'.$$

Note that both $T_1'T_3' = T_2'T_4'$ and $T_1'T_2' = T_3'T_4'$ are characterizations of tangential quadrilaterals. It was the first of these two that was proved in [18]. □