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Innovation Experiment Based on Circular Points and Laguerre Theorem in Computer Vision

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***Abstract*—The image coordinates of circular points calculation is one of the key technologies for camera self-calibration in computer vision. The definitions of planar circular points and isotropic line and Laguerre theorem are given firstly. Then according to the projective invariant of central projection, Laguerre theorem and the nature of the equilateral triangle, the image coordinates of vanishing points in sides of triangles and planar circular points are computed. At last an experiment of camera intrinsic parameters being determined based on the circular points is given. In this experiment, according to obtaining three different images and extract coordinates of every feature points are obtained, the image coordinates of vanishing points and circular points are obtained linearly, and camera intrinsic parameters can be determined through Choleskey decomposition. The result of this experiment shows the approach is effective.**

Keywords-computer vision; circular points; isotropic line; Laguerre theorem; equilateral triangle.

I. INTRODUCTION

In computer vision studies, there are important connections between the image of the absolute conic (IAC) and camera intrinsic parameters matrix[1]. Circular points are the specific points on the absolute conic, so the images of circular points have affinities with camera intrinsic parameters matrix. Since 2000, circular points are first introduced into camera calibration by Meng and others[2], we have seen the emergence of many camera self-calibration approaches based on circular points. So the researches into the approaches of computing image coordinates of circular points become a new hotspot of camera self-calibration in computer vision. Where some are based on the definition, solving image points of the intersection between the circle and the line at infinity and the intersection between any two circles on one plane, for example[3,4], however which are too computationally intensive, and others are based on the corollary to Laguerre theorem in projective geometry, solving image coordinates of circular points through the harmonic relation between the vanishing points which are orthogonal and the circular points, for example[5,6]. However the calibration process requires more stringent about the orthogonality of vanishing points.

On the basic of Laguerre theorem, this paper presents an approach to computing the image coordinates of circular points using the special angle relation of equilateral triangle. In order to verify the validity of this approach, at last a real experiment is given, in which the images coordinates of circular points are computed using this approach and the intrinsic parameters matrix are solved linearly. The experimental data shows the way is feasible.

II. PLANAR CIRCULAR POINTS

A. Cross Ratio and Harmonic Ratio

Cross-ratio is the basic invariant of central projection[7].

For the collinear four points P_1, P_2, P_3, P_4 , the cross-ratio (P_1P_2, P_3P_4) is the ratio of single ratio $(P_1P_2P_3)$ to single ratio $(P_1P_2P_4)$, that is

$$\begin{aligned}(P_1P_2, P_3P_4) &= (P_1P_2P_3)/(P_1P_2P_4)\\ &= \overline{P_1P_3}\cdot\overline{P_2P_4} : \overline{P_2P_3}\cdot\overline{P_1P_4}.\end{aligned} \quad (1)$$

Where P_1, P_2 are basic points, P_3, P_4 are segmentation points.

For the intersecting for lines p_1, p_2, p_3, p_4 , the cross-ratio (p_1p_2, p_3p_4) is

$$\begin{aligned}(p_1p_2, p_3p_4) &= \frac{(p_1p_2p_3)}{(p_1p_2p_4)}\\ &= \frac{\sin(p_1, p_3)\sin(p_2, p_4)}{\sin(p_2, p_3)\sin(p_1, p_4)}.\end{aligned} \quad (2)$$

Where p_1, p_2 are basic lines, p_3, p_4 are split lines.

If $(P_1P_2, P_3P_4) = -1$, the points P_3, P_4 separate harmonically the points P_1, P_2, or rather the points P_1, P_2 and the points P_3, P_4 are harmonic conjugates or P_4 is the fourth harmonic point, where the value -1of cross-ratio is called harmonic ratio[7].

We may verify easily that the two end points of a segment are separated harmonically by its midpoint and the point at infinity.

B. Circular Points and Its Images

Defining circular points, complex conjugate points $I(1,i,0)$ and $J(1,-i,0)$ are called circular points[7].

Let Π be a finite plane in space. Select any two mutually orthogonal lines as x axis and y axis, o is the intersection of x axis and y axis, then one can get a right-handed WCS $o-xyz$, where the z axis passes the point o and it is orthogonal with the plane Π , thus the equation of Π is $z=0$. In the homogeneous coordinate system of projective plane, the line at infinity on the plane Π can be expressed as:

$$\begin{cases} w=0 \\ z=0 \end{cases}. \tag{3}$$

Let C be any circle, which center is $(x_0 \quad y_0 \quad 0 \quad 1)^T$ in the homogeneous coordinates and radius is r , so the equations of the circle is

$$\begin{cases} (x-x_0w)^2+(y-y_0w)^2=w^2r^2 \\ z=0 \end{cases}. \tag{4}$$

It is easy to verify that the circular points of the plane are the intersection of the line at infinity and any circle on the same plane, and they are on the absolute conic $(X^TX=0)$ on the plane at infinity. Using the knowledge of projective geometry, their images are the intersection of the image of line at infinity and the image of the circle in the imaging plane. Because the elements remains the same nature under the real projective transformation, then the images of circular points are a pair of conjugate imaginary images, denoted by

$$m_i(x_r+x_ii \quad y_r+y_ii \quad 1),$$
$$m_j(x_r-x_ii \quad y_r-y_ii \quad 1).$$

C. Isotropic Line

Defining isotropic line, the line through the circular points is called isotropic line if it is not the line at infinity[5].

The isotropic line is also called line of the minimum, obviously, there are two non-isotropic lines through any point P and the intersection point of any non-isotropic line and the line at infinity is circular point.

D. Laguerre Theorem

l_1,l_2 are any two non- isotropic lines and l_i,l_j are two isotropic lines which through the intersection point of l_1 and l_2 . Let the angle of intersection of l_1,l_2 be θ and the value of cross-ration of l_1,l_2,l_i,l_j be μ , then there are

$$\mu=e^{2i\theta}, \tag{5}$$

or

$$\theta=\ln\mu/2i. \tag{6}$$

III. LAGURRE THEOREM APPLICATIONS

Shown in Figure1, D,E,F are midpoints of the edges BC,AC,AB in triangle ABC respectively, and Figure2 is an imaging plane of figure1. Using Laguerre theorem solve the image coordinates of circular points in plane ABC .

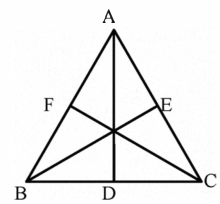

Figure 1. Planar triangle

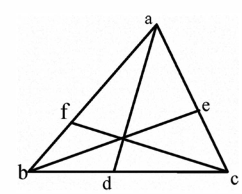

Figure 2. The image of triangle

A. Solving Vannishing Points of Edges of the Triangle

Shown in Figure1, the points at infinity of AB,AC,BC are $P_{1\infty},P_{2\infty},P_{3\infty}$. Points a,b,c,d,e,f,p_1,p_2,p_3 are the corresponding points on the imaging plane of points $A,B,C,D,E,F,P_{1\infty},P_{2\infty}$, $P_{3\infty}$ respectively, and their coordinates are (u_a,v_a) , (u_b,v_b), (u_c,v_c) , (u_d,v_d), (u_e,v_e), (u_f,v_f), (u_{p1},v_{p1}) ,(u_{p2},v_{p2}),(u_{p3},v_{p3}).

Now, take the edge AB into account, according to the projective invariant of central projection, there are

$$\begin{cases} (ab,fp_1)=-1 \\ ab\times ap_1=0 \end{cases}. \tag{7}$$

Equation (7) can be expressed by matrix as follows:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ v_a - v_b & u_b - u_a \end{bmatrix} \begin{bmatrix} u_{p1} \\ v_{p1} \end{bmatrix} = \begin{bmatrix} \left(u_f\left(u_a + u_b\right) - 2u_a u_b\right)/\left(2u_f - u_a - u_b\right) \\ \left(v_f\left(v_a + v_b\right) - 2v_a v_b\right)/\left(2v_f - v_a - v_b\right) \\ u_b v_a - u_a v_b \end{bmatrix}. \quad (8)$$

The coordinate of p_1 can be solved by the least-squares method. Similarly, the coordinates of the vanishing points p_2, p_3 can be gotten.

B. Solving Image Coordinates of Circular Points

Let p_1, p_2, p_3 be the vanishing points of edges ab, ac and bc, according to the Laguerre theorem and the nature of the equilateral triangle, the images coordinates of circular points m_i, m_j could be calculated. The process is as follows:

On the plane of ABC, there are two isotropic lines l_{AB}, l_{AC} and two non-isotropic lines l_i, l_j pass through the point A, and according to (8), one has

$$\begin{cases} \dfrac{(x_r + x_i i) - u_{p1}}{(x_r + x_i i) - u_{p2}} \cdot \dfrac{(x_r - x_i i) - u_{p2}}{(x_r - x_i i) - u_{p1}} = \cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3} \\ \dfrac{(y_r + y_i i) - v_{p1}}{(y_r + y_i i) - v_{p2}} \cdot \dfrac{(y_r - y_i i) - v_{p2}}{(y_r - y_i i) - v_{p1}} = \cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3} \end{cases}, \quad (9)$$

In (9), the real–part and imaginary part on both side of the equations are always equal, so the equations can be obtained as follows:

$$\begin{cases} \sqrt{3}\left(x_r^2 + x_i^2\right) - \sqrt{3}\left(u_{p1} + u_{p2}\right)x_r - \left(u_{p1} - u_{p2}\right)x_i = -\sqrt{3}u_{p1}u_{p2} \\ \sqrt{3}\left(y_r^2 + y_i^2\right) - \sqrt{3}\left(v_{p1} + v_{p2}\right)y_r - \left(v_{p1} - v_{p2}\right)y_i = -\sqrt{3}v_{p1}v_{p2} \end{cases}. \quad (10)$$

Similarly, taking points B, C into account, one has

$$\begin{cases} \sqrt{3}\left(x_r^2 + x_i^2\right) - \sqrt{3}\left(u_{p2} + u_{p3}\right)x_r - \left(u_{p3} - u_{p3}\right)x_i = -\sqrt{3}u_{p2}u_{p3} \\ \sqrt{3}\left(y_r^2 + y_i^2\right) - \sqrt{3}\left(v_{p2} + v_{p3}\right)y_r - \left(v_{p2} - v_{p3}\right)y_i = -\sqrt{3}v_{p2}v_{p3} \end{cases}, \quad (11)$$

and

$$\begin{cases} \sqrt{3}\left(x_r^2 + x_i^2\right) - \sqrt{3}\left(u_{p3} + u_{p1}\right)x_r - \left(u_{p3} - u_{p1}\right)x_i = -\sqrt{3}u_{p3}u_{p1} \\ \sqrt{3}\left(y_r^2 + y_i^2\right) - \sqrt{3}\left(v_{p3} + v_{p1}\right)y_r - \left(v_{p3} - v_{p1}\right)y_i = -\sqrt{3}v_{p3}v_{p1} \end{cases}. \quad (12)$$

Simultaneous system of the foregoing equations, one has

$$\begin{bmatrix} \sqrt{3}\left(u_{p1} - u_{p3}\right) & u_{p1} + u_{p3} - 2u_{p2} \\ \sqrt{3}\left(u_{p3} - u_{p2}\right) & u_{p2} + u_{p3} - 2u_{p1} \end{bmatrix} \begin{bmatrix} x_r \\ x_i \end{bmatrix} = \begin{bmatrix} \sqrt{3}\left(u_{p1}u_{p2} - u_{p2}u_{p3}\right) \\ \sqrt{3}\left(u_{p3}u_{p1} - u_{p1}u_{p2}\right) \end{bmatrix}, \quad (13)$$

and

$$\begin{bmatrix} \sqrt{3}\left(v_{p1} - v_{p3}\right) & v_{p1} + v_{p3} - 2v_{p2} \\ \sqrt{3}\left(v_{p3} - v_{p2}\right) & v_{p2} + v_{p3} - 2v_{p1} \end{bmatrix} \begin{bmatrix} y_r \\ y_i \end{bmatrix} = \begin{bmatrix} \sqrt{3}\left(v_{p1}v_{p2} - v_{p2}v_{p3}\right) \\ \sqrt{3}\left(v_{p3}v_{p1} - v_{p1}v_{p2}\right) \end{bmatrix}. \quad (14)$$

Then the image coordinates of dual circular points

$$m_i\left(x_r + x_i i \quad y_r + y_i i \quad 1\right),$$

$$m_j\left(x_r - x_i i \quad y_r - y_i i \quad 1\right)$$

can be solved from foregoing (13) and (14).

IV. EXPERIMENT

A. Solving Camera Intrinsic Parameters

In this experiment, adopted camera model is the usual pinhole model, camera mathematical model is

$$\lambda_c m = K\left[R \quad T\right]M, \quad (15)$$

where $m(u,v,1)^T$ is the homogeneous coordinate vector of a 2D point in image. $M(x,y,z,1)^T$ is the homogeneous coordinate vector of a 3D point in object space. λ_c is an arbitrary scale factor; $K = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$ is a 3×3 camera intrinsic parameters matrix; R is a 3×3 orthogonal ration matrix; T is a 3×1 translation vector.

Due to m_i, m_j that are the image points of I, J being a pair of conjugate points on the IAC [8], then according to the camera model, only two constrain equations on camera intrinsic parameters can be gotten as follows:

$$\begin{cases} \mathrm{Re}(m_i^T \omega m_i) = 0 \\ \mathrm{Im}\left(m_i^T \omega m_i\right) = 0 \end{cases}, \quad (16)$$

where Re, Im denote real–part and imaginary part respectively. Then according to (16), as long as 3(or more) images, ω can be solved linearly, and five intrinsic parameters can be obtained by taking ω Choleskey decomposition.

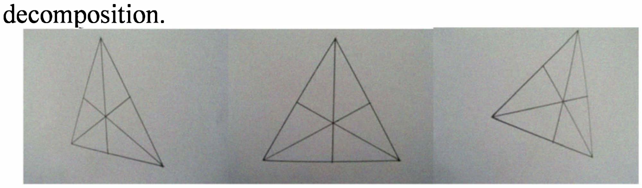

Figure 3. Real experiment images

We take 3 photos from different orientations, shown as Figure3. In order to verify the validity of this approach, a real experiment is given. For each photo, firstly the image coordinates of every feature are extracted using Harris corner operator in [9], that are (u_a, v_a), (u_b, v_b), (u_c, v_c), (u_d, v_d), (u_e, v_e), (u_f, v_f). Then coordinates of vanishing points in every edges of triangles are computed by (8), denoted by (u_{pi}, v_{pi}) ($i = 1,2,3$), and the image coordinates of circular points are obtained by (13) and (14). At last, ω can be solved by (16), and intrinsic parameters matrix K can be obtained by taking ω Choleskey decomposition.

The result of the experiment is f_u =740.4681, f_v =724.8945, s =-10.843, u_0 =301.1129, v_0 =208.0893. And using the same photos, according to the approach in[10], the result is f_u =734.9644, f_v =723.2361, s =43.8469, u_0 =276.6817, v_0 =239.3070. Comparisons of the results of the two approaches show the way is feasible.

V. CONCUSION

This paper presents an approach to compute the image coordinates of circular points using Laguerre theorem based on the special angle relation of equilateral triangle. This approach is on the grounds of the basic Laguerre theorem and makes full use of triangle simply geometry features. And Laguerre theorem and circular points are used in camera calibration in computer vision from a new point of view.

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